Q1

1

The "difference of two squares" says that $a^2 - b^2$ factorises to $(a + b)(a - b)$
Both terms are not squares so take out a common factor of 5

$$5(4x^2 - 1)$$

[1]

Write the terms inside the brackets as squares

$$5((2x)^2 - 1^2)$$

Use the "difference of two squares" to factorise inside the brackets

$$5((2x + 1)(2x - 1))$$

The outer pair of brackets are no longer necessary as all terms are being multiplied (for example 5×(4×3) is the same as 5×4×3)

$\mathbf{5(2x + 1)(2x - 1)}$ [1]

Q2

2a

The "difference of two squares" says that $a^2 - b^2$ factorises to $(a + b)(a - b)$
Write both terms as squares

$$y^2 - 4^2$$

Use the "difference of two squares" to factorise

$\mathbf{(y + 4)(y - 4)}$ [1]

2b

Multiply 2 by -10

$$2 \times -10 = -20$$

Find two numbers that multiply to give -20 and add to give -1 (the middle number)

-5 and 4

Split the middle term into $-5p$ and $4p$

$$2p^2 - 5p + 4p - 10$$

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$$2p^2 - 5p + 4p - 10$$

Factorise fully the first pair of terms and the second pair of terms

$$p(2p - 5) + 2(2p - 5)$$

Factorise out the whole $(2p - 5)$ bracket as if it were one term, leaving p and $+2$ in their own bracket

$$(2p - 5)(p + 2)$$

allowing sign errors [1]
$\mathbf{(2p - 5)(p + 2)}$ [1]

Q3

3

The "difference of two squares" says that $a^2 - b^2$ factorises to $(a + b)(a - b)$
Write both terms as squares

$$(2x)^2 - 3^2$$

Use the "difference of two squares" to factorise

$$\mathbf{(2x + 3)(2x - 3)}\ [1]$$

Q4

4a

The "difference of two squares" says that $a^2 - b^2$ factorises to $(a + b)(a - b)$

$$\mathbf{(a + b)(a - b)}\ [1]$$

4b

Method 1
Following the "hence" (using what was done before), let $a = x^2 + 4$ and $b = x^2 - 2$

$$a^2 - b^2$$

[1]

Use the "difference of two squares" to factorise

$$(a + b)(a - b)$$

Replace a with $x^2 + 4$ in brackets and b with $x^2 - 2$ in brackets

$$((x^2 + 4) + (x^2 - 2))((x^2 + 4) - (x^2 - 2))$$

Expand inside the brackets

$$(x^2 + 4 + x^2 - 2)(x^2 + 4 - x^2 + 2)$$

[1]

$$(x^2+4+x^2-2)(x^2+4-x^2+2)$$

[1]

Collect "like" terms

$$(2x^2+2)(6)$$

Multiply out the brackets (it may help to write the 6 at the front)

$12x^2 + 12$ [1]

Method 2
Following the "or otherwise", write out the squared brackets as the product of two brackets

$$(x^2+4)(x^2+4)-(x^2-2)(x^2-2)$$

Multiply out the two sets of brackets and collect like terms (keeping the results in their own separate brackets)

$$(x^4+4x^2+4x^2+16)-(x^4-2x^2-2x^2+4)$$
$$=(x^4+8x^2+16)-(x^4-4x^2+4)$$

[1]

Remove the brackets (by multiplying all terms by -1 in the second bracket)

$$x^4+8x^2+16-x^4+4x^2-4$$

[1]

Remove the brackets (by multiplying all terms by -1 in the second bracket)

$$x^4+8x^2+16-x^4+4x^2-4$$

[1]

Collect like terms

$12x^2 + 12$ [1]

Q5

5

There is a lot going on in this question so deal with numbers first, then letters (in order they appear, usually alphabetically).

15 and 35 have a common factor of 5 so 5 will be "outside the bracket"

There are b's in both terms - the lowest power of which is b^3.

b^3 will be outside the bracket

The lowest power of c's is c.

c will be outside the bracket

Now construct the factorised answer by consider what is left by dividing each term by each factor listed above.

$$15 \div 5 = 3 \text{ and } 35 \div 5 = 7$$
$$b^5 \div b^3 = b^2 \text{ and } b^3 \div b^3 = 1$$
$$c \div c = 1 \text{ and } c^9 \div c = c^8$$

and so

$15b^5c - 35b^3c^9 = 5b^3c(3b^2 - 7c^8)$
Partially factorised [1]
Fully factorised [1]

Q6

6

Deal with numbers first, then letters (in order they appear, usually alphabetically).

15 and 20 have a common factor of 5 so 5 will be "outside the bracket"

There are y's in both terms - the lowest power of which is y^3.

y^3 will be outside the bracket

Be careful with the u - this only appears in one term so cannot be factorised.
Now construct the factorised answer by consider what is left by dividing each term by each factor listed above.

$$15 \div 5 = 3 \text{ and } 20 \div 5 = 4$$
$$y^4 \div y^3 = y \text{ and } y^3 \div y^3 = 1$$

and so

$$\mathbf{15y^4 + 20uy^3 = 5y^3(3y + 4u)}$$
Partially factorised [1]
Fully factorised [1]

Q7

7

There is a lot going on in this question so deal with numbers first, then letters (in order they appear, usually alphabetically).

25 and 45 have a common factor of 5 so 5 will be "outside the bracket"

There are a's in both terms - the lowest power of which is a^4.

a^4 will be outside the bracket

The lowest power of c's is c^3.

c^3 will be outside the bracket

Be careful with the d and h - both of thesse occur in one term only so cannot be factorised.
Construct the factorised answer by consider what is left by dividing each term by each factor listed above.

$$25 \div 5 = 5 \text{ and } 45 \div 5 = 9$$
$$a^4 \div a^4 = 1 \text{ and } a^9 \div a^4 = a^5$$
$$c^7 \div c^3 = c^4 \text{ and } c^3 \div c^3 = 1$$

and so

$$\mathbf{25a^4c^7d + 45a^9c^3h = 5a^4c^3(5c^4d + 9a^5h)}$$
Two letters factorised [1]
Fully factorised [1]

Q8-9

8

We can see straight away there is a factor of 2 in both terms.

$$2e^2 - 18 = 2(e^2 - 9)$$

[1]

Spot that $e^2 - 9$ is the difference of two squares.

$$2(e^2 - 9) = 2(e + 3)(e - 3)$$

There is nothing else that can be factorised.

$$\mathbf{\therefore 2e^2 - 18 = 2(e + 3)(e - 3)}$$ [1]

9

Spotting that 4 and 9 are square numbers leads us to this being the difference of two squares.

$$4c^2 - 9d^2 = (2c)^2 - (3d)^2$$

So

$$(2c)^2 - (3d)^2 = (2c + 3d)(2c - 3d)$$

$$\mathbf{4c^2 - 9d^2 = (2c + 3d)(2c - 3d)}$$ [1]

Q10

10

Recognise this is a quadratic expression and is non-monic ($a \neq 1$), so use factorising "by grouping".
Find the product "ac".

$$ac = 6 \times (-5) = -30$$

Consider "b".

$$b = -1$$

We need two numbers whose product is -30 and sum is -1.

$$-6 \text{ and } 5$$

Rewrite the middle term using these values.

$$6y^2 - y - 5 = 6y^2 - 6y + 5y - 5$$

Factorise the first pair of terms and the second pair of terms.

$$6y^2 - 6y + 5y - 5 = 6y(y-1) + 5(y-1)$$

[1]

Now factorise $(y-1)$ from each of these terms.

$$6y(y-1) + 5(y-1) = (y-1)(6y+5)$$

$\mathbf{6y^2 - y - 5 = (y-1)(6y+5)}$ [1]

Q11

Multiply 3 by -20 (the first and last numbers)

$$3 \times -20 = -60$$

Find two numbers that multiply to give -60 and add to give 11 (the middle number)

15 and -4

Split the middle term into $15x$ and $-4x$

$$3x^2 + 15x - 4x - 20$$

Factorise fully the first pair of terms and the second pair of terms

$$3x(x+5) - 4(x+5)$$

or $x(3x - 4) + 5(3x - 4)$ if terms written other way round [1]

Factorise out the whole (x + 5) bracket as if it were one term, leaving $3x$ and -4 in their own bracket

$$(x+5)(3x-4)$$

$\mathbf{(x+5)(3x-4)}$ [1]

Q12

12

The coefficient of the x^2 term is not 1 so you can factorise the expression by grouping and factorising.

Find the product of the coefficient of the x^2 term and the constant term.

$$3 \times 8 = 24$$

Find the factor pairs of this product.

$$1, 24 \text{ or } -1, -24$$
$$2, 12 \text{ or } -2, -12$$
$$3, 8 \text{ or } -3, -8$$
$$4, 6 \text{ or } -4, -6$$

Find the factor pair that adds together to give you the same value as the coefficient of the x term.

$$2, 12: \quad 2 + 12 = 14$$

Rewrite the original expression splitting up the x term using the values determined above.

$$3x^2 + 2x + 12x + 8$$

Group the terms into two pairs, the first two terms as one pair and the last two terms as the second pair.
Factorise both pairs of terms.

$$x(3x + 2) + 4(3x + 2)$$

Both terms in the expression have a factor of $(3x + 2)$, so can now be factorised with this as the common factor.

$$\mathbf{(3x + 2)(x + 4)}$$

Factorisation to $(3x + a)(x + b)$ *where* $ab = 8$ *or* $a + 3b = 14$ [1]
Fully correct factorisation [1]